Upper Bounds on the Lifetime of Sensor Networks

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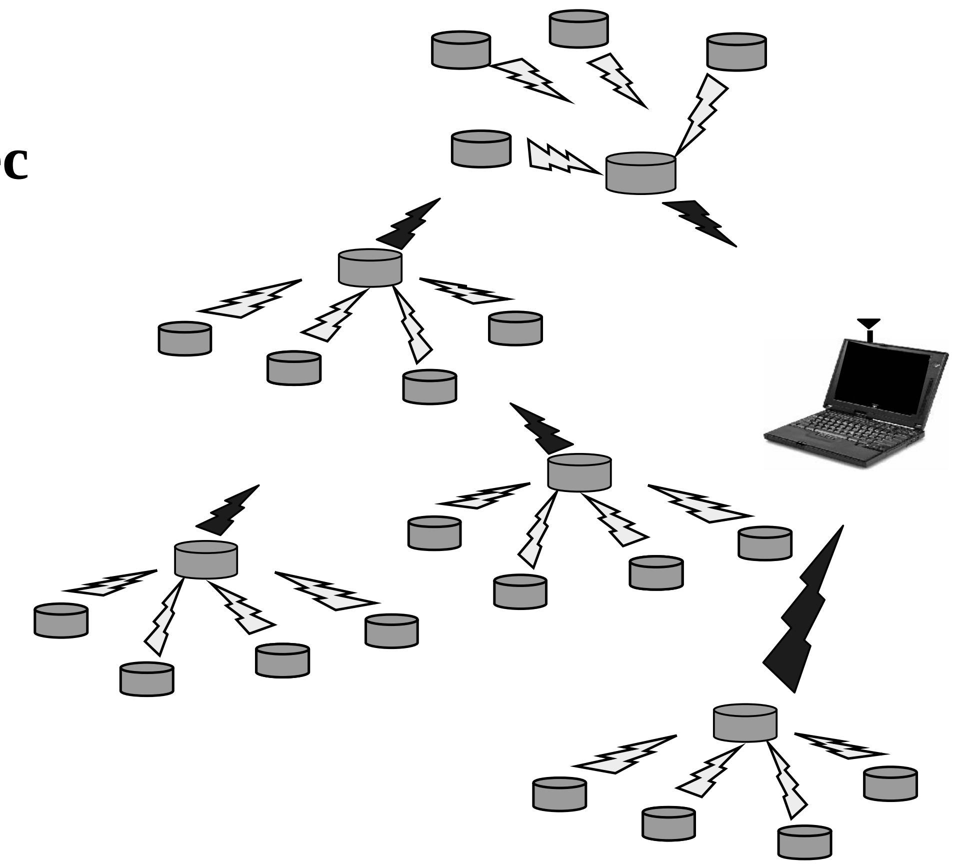
Outline

- Wireless Sensor Networks
- Energy Models
- The Lifetime Problem
- Bounding Lifetime
- Extensions
- Summary

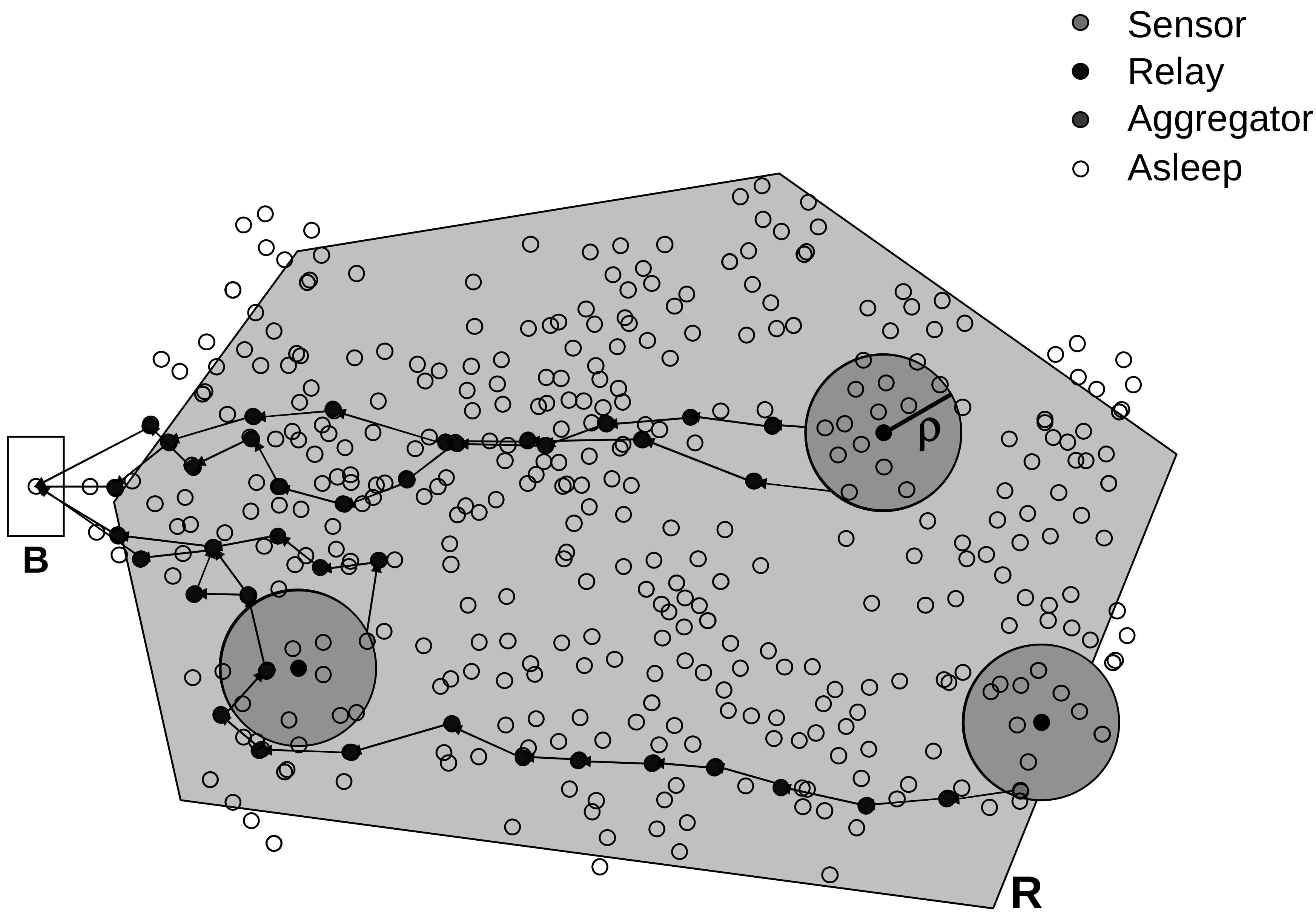
Wireless Sensor Networks

- **Sensor Types: Low Rate (e.g., acoustic and seismic)**
- **Bandwidth: bits/sec to kbits/sec**
- **Transmission Distance: 5-10m (< 100m)**
- **Spatial Density**
  - **0.1 nodes/m^2 to 20 nodes/m^2**
- **Node Requirements**
  - **Small Form Factor**
  - **Required Lifetime: > year**

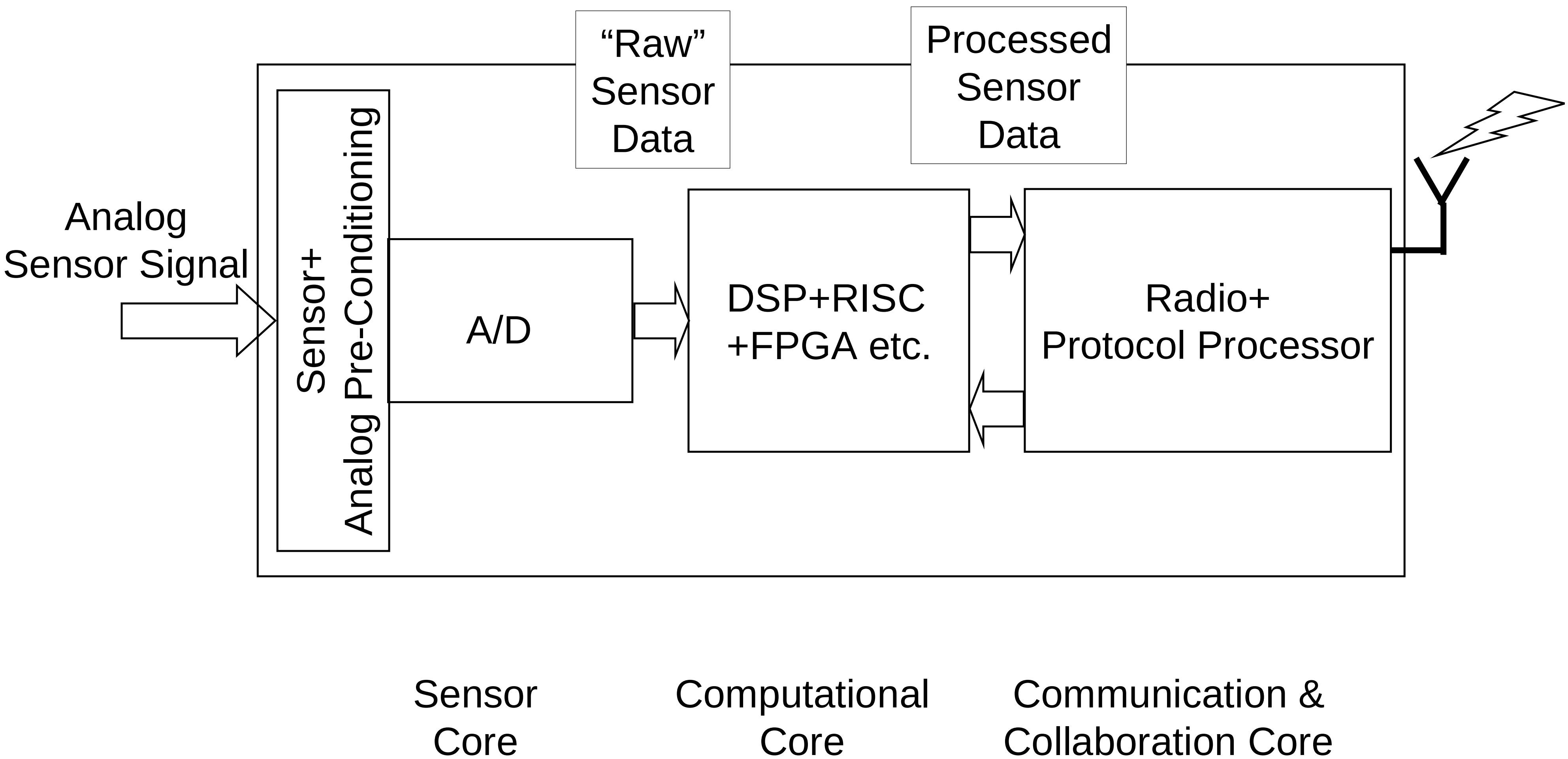


Key Challenge: Maximizing Lifetime

Data Gathering Wireless Networks: A Primer



Functional Abstraction of DGWN Node



Energy Models

d

Transmit Energy Per Bit $E_{tx} = \alpha_{11} + \alpha_2 d^n$ n = Path loss index

1. Transceiver Electronics
2. Startup Energy

Power-Amp

Receive Energy Per Bit $E_{rx} = \alpha_{12}$

d

Relay Energy Per Bit $E_{relay} = \alpha_{11} + \alpha_2 d^n + \alpha_{12} = \alpha_1 + \alpha_2 d^n$

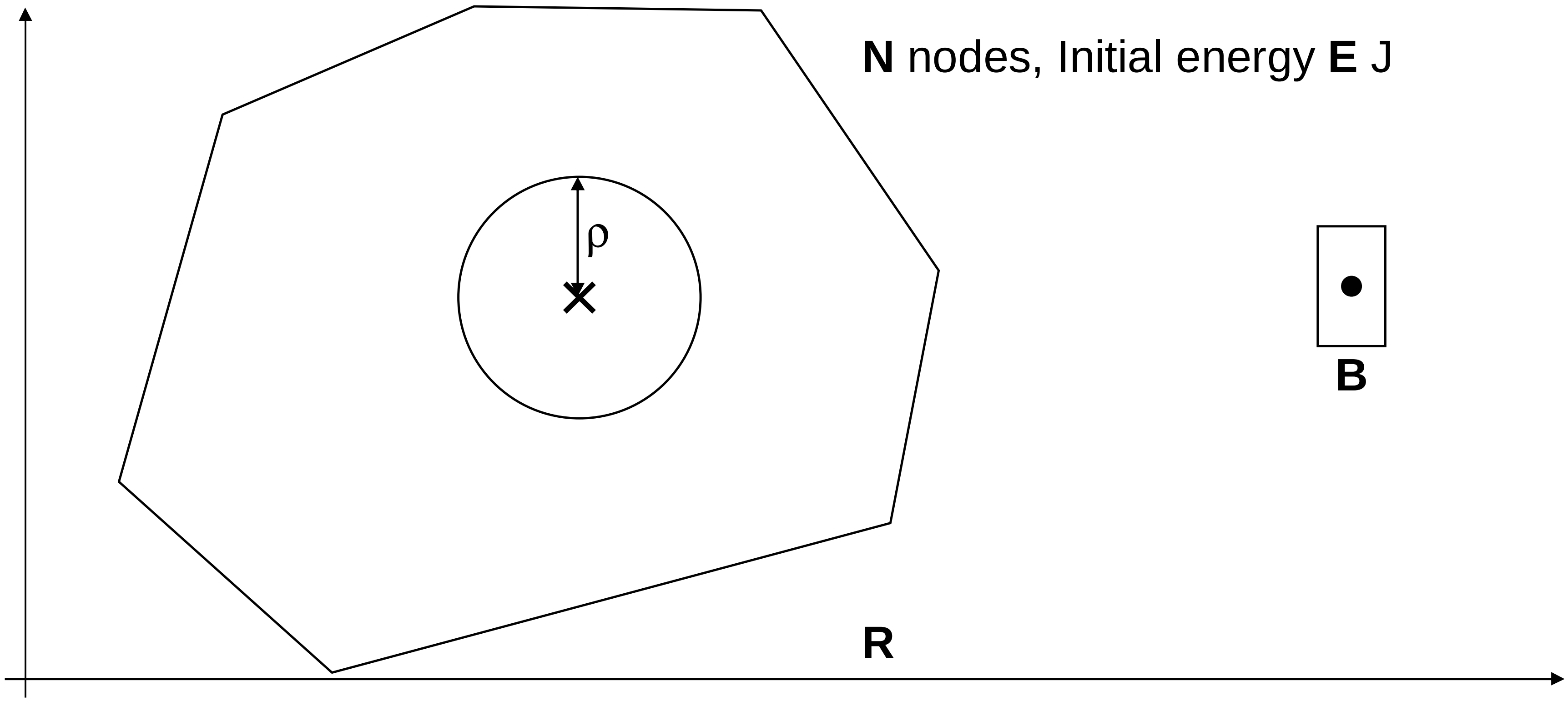
$P_{relay} = (\alpha_1 + \alpha_2 d^n) r$

Sensing Energy Per Bit $E_{sense} = \alpha_3$

Defining Lifetime

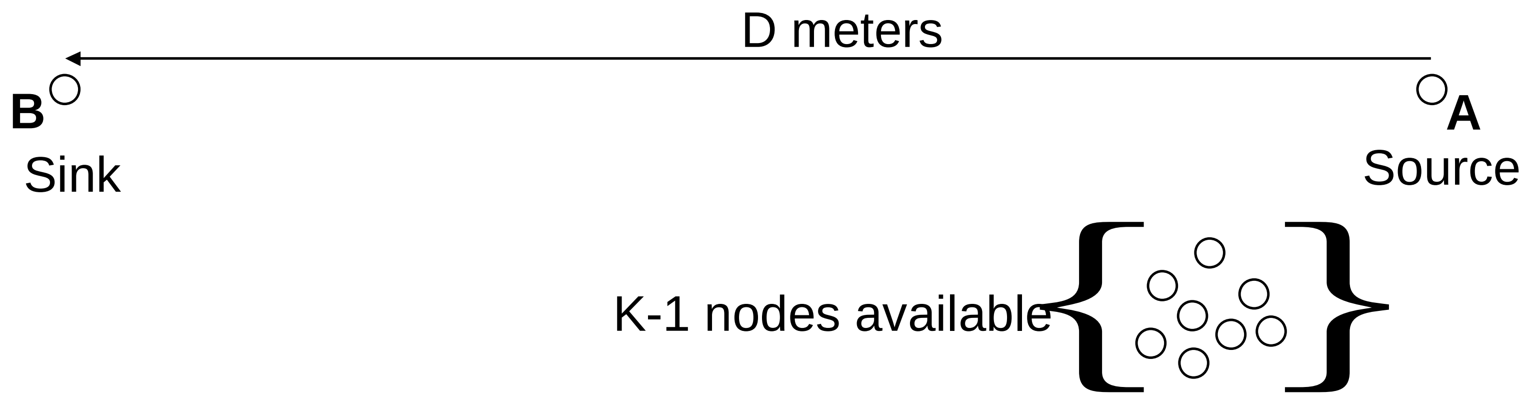
- Three network states:
  - Active
  - Failure
  - Dormant
- Possible lifetime definitions:
  - Cumulative active time
  - Cumulative active time to first failure

The Lifetime Bound Problem



- **Bound the lifetime of a network given:**
  - **A description of R and the relative position of the base-station**
  - **The number of nodes (N) and initial energy in each node (E)**
  - **Node energy parameters (α_1, α_2, α_3), path loss index *n***
  - **Source observability radius (ρ)**
  - **Spatial distribution of the source ($l_{source}(x,y)$)**
  - **Expected source rate (*r* bps)**
- **Note: Bound is topology insensitive**

Preliminaries: Minimum-Energy Links and Characteristic Distance

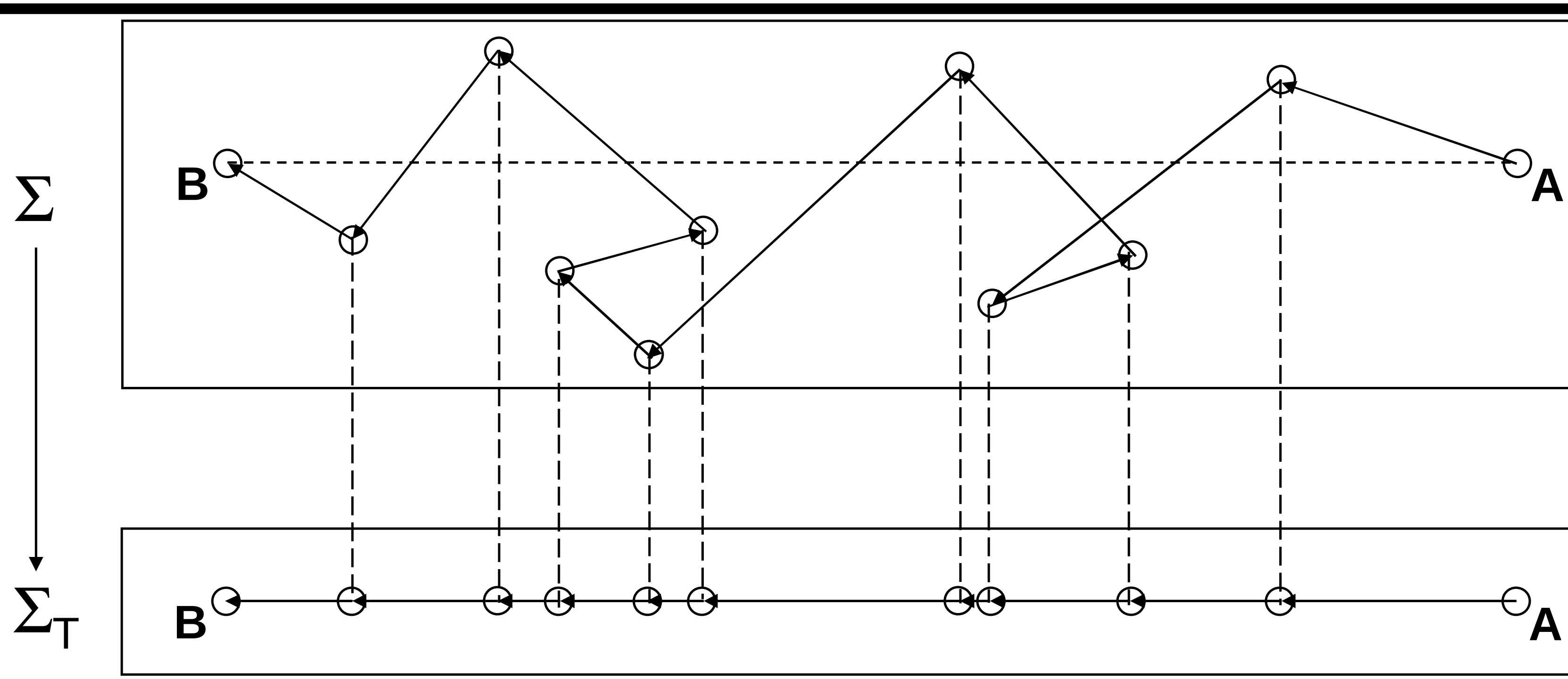


- **Given:** A source and sink node *D* m apart and *K-1* available nodes that act as relays and can be placed at will (a relay is qualified by its source and destination)
- **Solution:** Position, qualification of the K-1 relays
- **Measure of the solution:** Energy needed to transport a bit or equivalently, the total power of the link –

$$P_{\mathrm{link}}(D) = -\alpha_{12} + \sum_{i=1}^{K} P_{\mathrm{relay}}(d_i)$$

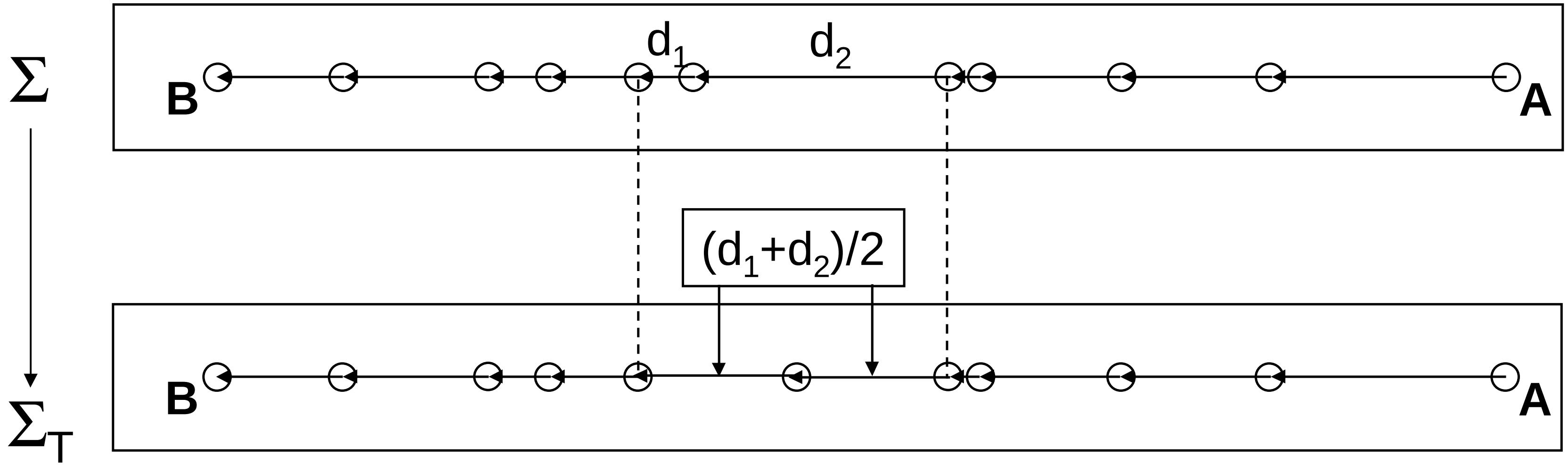
- **Problem:** Find a solution that minimizes the measure

Claim I: Optimal Solution is Collinear w/ Non-Overlapping Link Projections



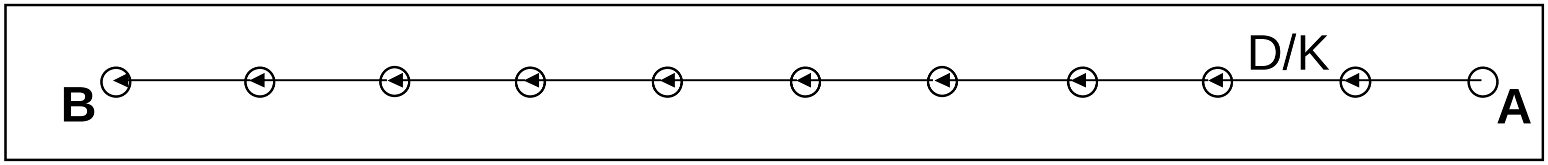
- **Proof: By contradiction. Suppose a non-compliant solution Σ is optimal**
- **Produce another solution Σ_T via the projection transformation shown**
- **Trivial to prove that measure(Σ_T) < measure(Σ) (QED)**
- **Result holds for any radio function monotonic in d**
- **Reduces to a 1-D problem**

Claim II: Optimal Solution Has Equal Hop Distances



- **Proof: By contradiction. Suppose a non-compliant solution Σ is optimal**
- **Produce solution Σ_T by taking any two unequal adjacent hops in Σ and making them equal to half the total hop length**
- **For any convex $P_{relay}(d)$, measure(Σ_T) < measure(Σ) (recall that $2f((x_1+x_2)/2) < f(x_1)+f(x_2)$ for a convex function f) (QED)**

Optimal Solution



- Measure of the optimal solution: $-\alpha_{12}+KP_{relay}(D/K)$
- P_{relay} convex $\Rightarrow KP_{relay}(D/K)$ is convex
- The continuous function $xP_{relay}(D/x)$ is minimized when:

$$x = \frac{D}{\sqrt[n]{\frac{\alpha_1}{\alpha_2(n-1)}}} = \frac{D}{D_{char}} \quad \Rightarrow \quad \min_x xP_{\text{relay}}\left(\frac{D}{x}\right) = \frac{n\alpha_1}{n-1}\frac{D}{D_{char}}$$

- Hence, the K that minimizes $P_{link}(D)$ is given by:

$$K_{opt} = \left\lceil \frac{D}{D_{char}} \right\rceil \text{ or } \left\lfloor \frac{D}{D_{char}} \right\rfloor \quad \Rightarrow \quad P_{\text{link}}(D) \geq \left(\frac{n\alpha_1}{n-1}\frac{D}{D_{char}} - \alpha_{12} \right) r$$

Corollary: Minimum Energy Relay

- **It is not possible to relay bits from A to B at a rate r using total link power less than:**

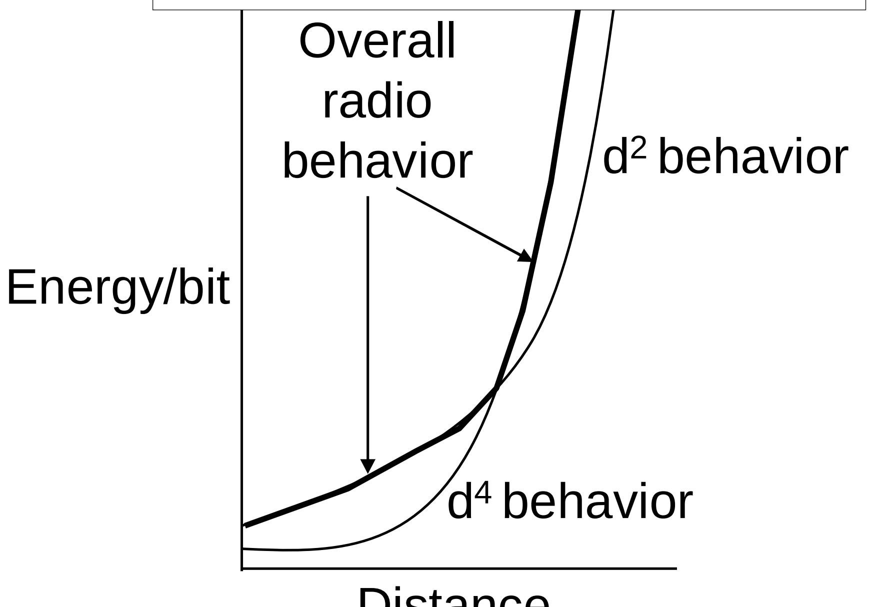
$$P_{\text{link}}(D) \geq \left(\frac{n\alpha_1}{n-1} \frac{D}{D_{char}} - \alpha_{12} \right) r$$

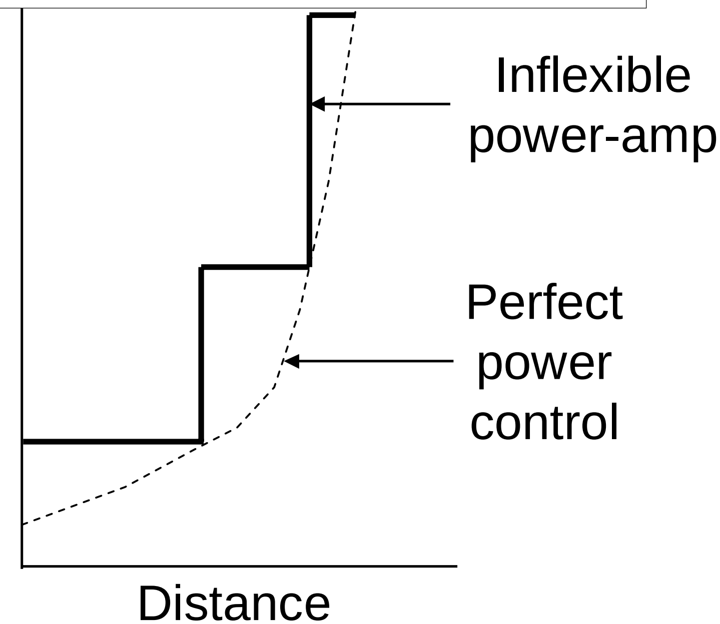
with equality ⇔ D is an integral multiple of D_{char}

- **Key points:**
  - **It is possible to relay bits with an energy cost linear in distance, regardless of the path loss index, n**
  - **The most energy efficient multi-hop links result when nodes are placed D_{char} apart**

Digression: Practical Radios

- **Results hinge only on communication energy versus distance being monotonically increasing and convex**



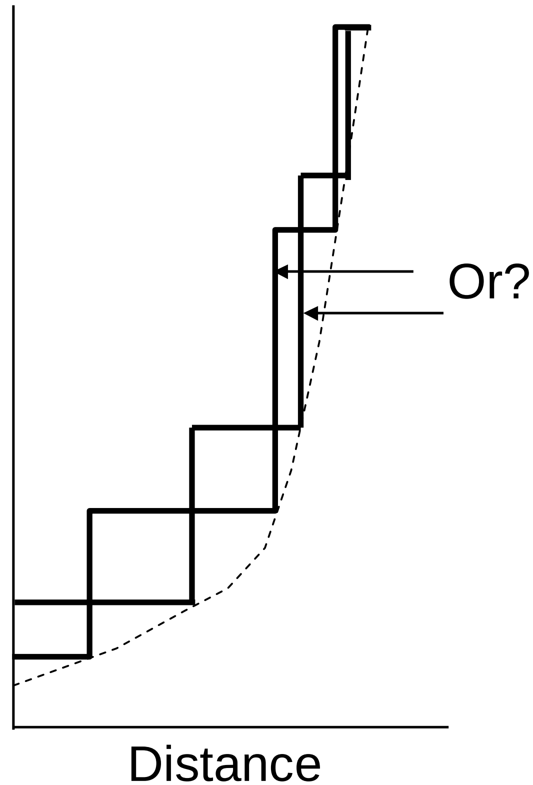


Complex path loss behavior
- Not a problem!
- Energy/bit can be made linear
- Equal hops still best strategy
- But … D_{char} varies with distance

Finite Power-Control Resolution
- “Too Coarse” quanta a problem
- Energy/bit no longer linear
- Equal hops NOT best for energy
- No concept of D_{char}

Digression: The Optimum Power-Control Problem

- **What is the best way to quantize the radio energy curve(for a given number of levels)?**

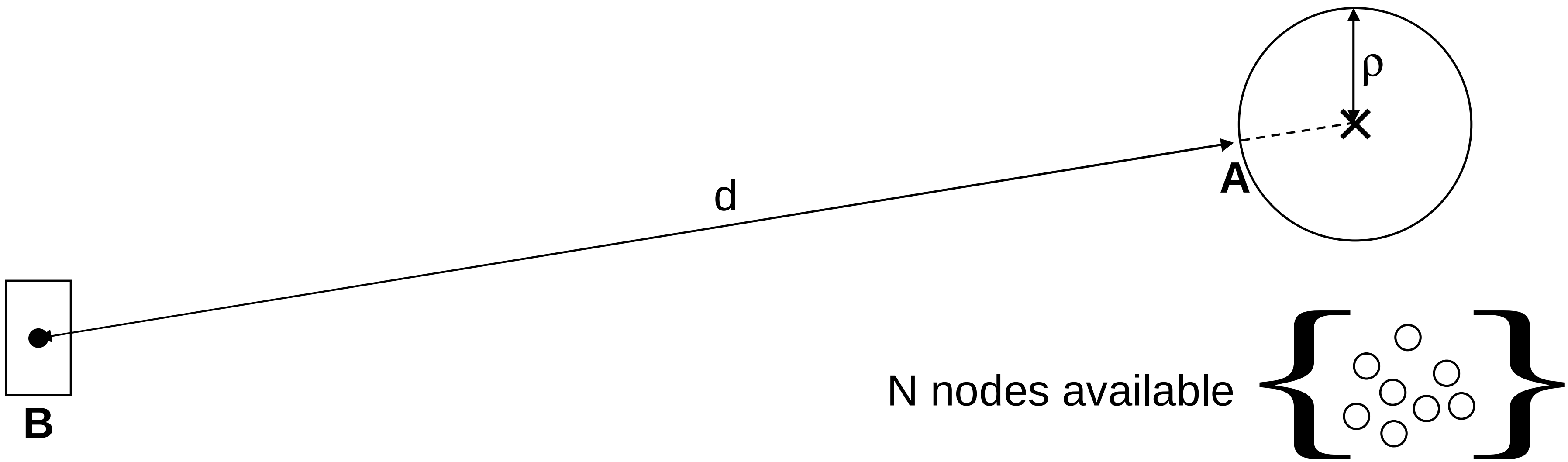


Answer depends on:

• Distribution of distances

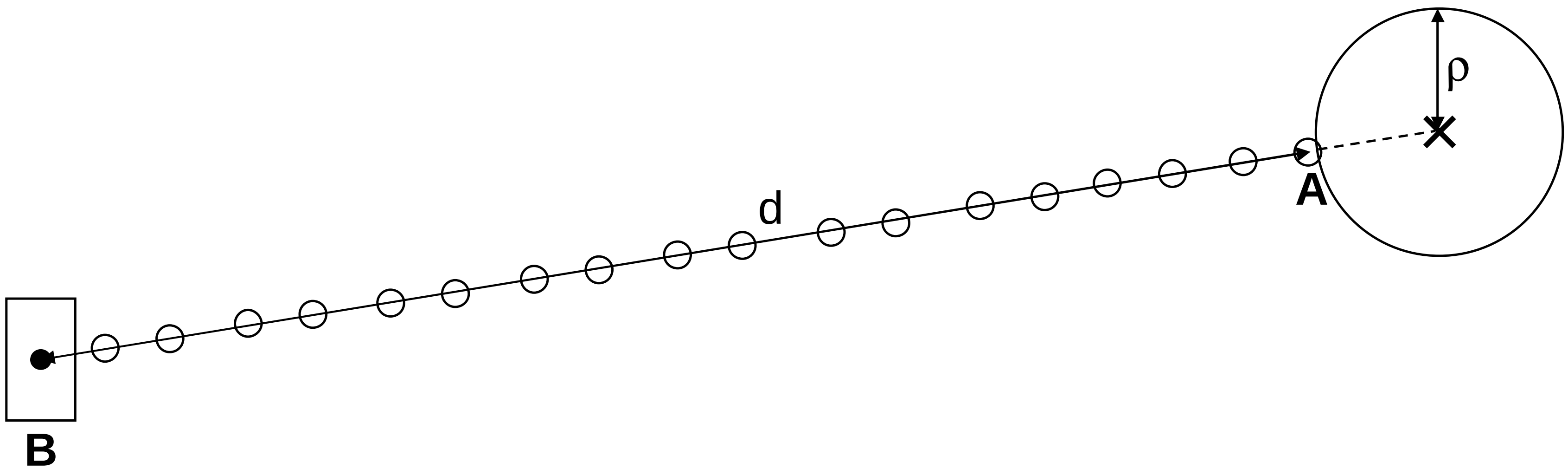
• Sophisticated non-linear optimization needed for best multi-hop

Maximizing Lifetime – A Simple Case

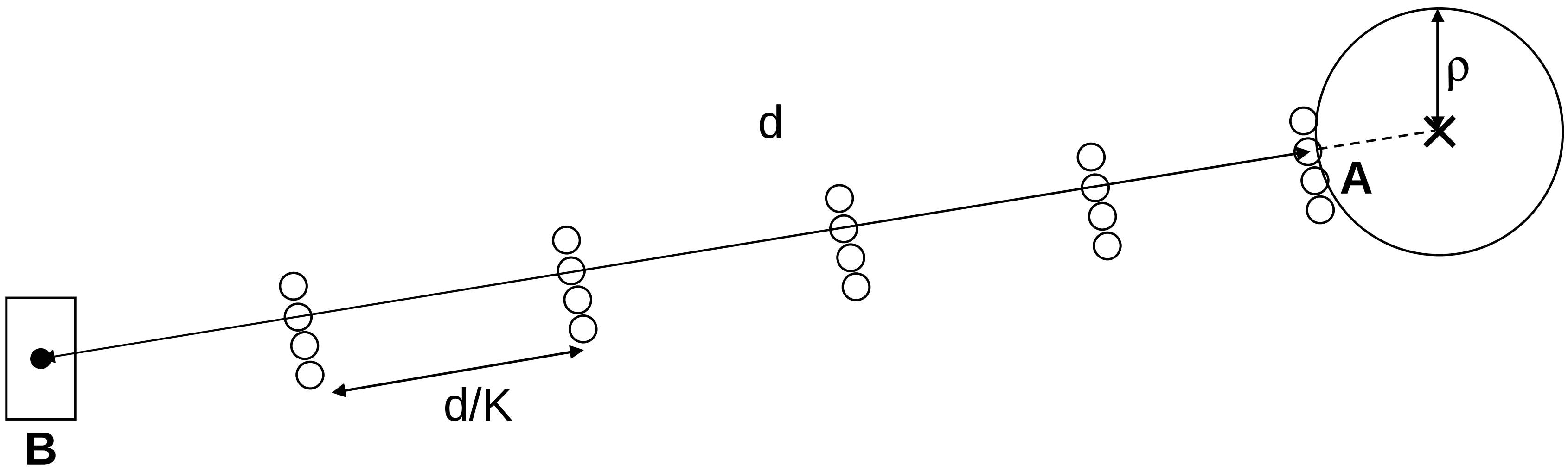


- **Problem: Using N nodes what is maximum sensing lifetime one can ever hope to achieve?**

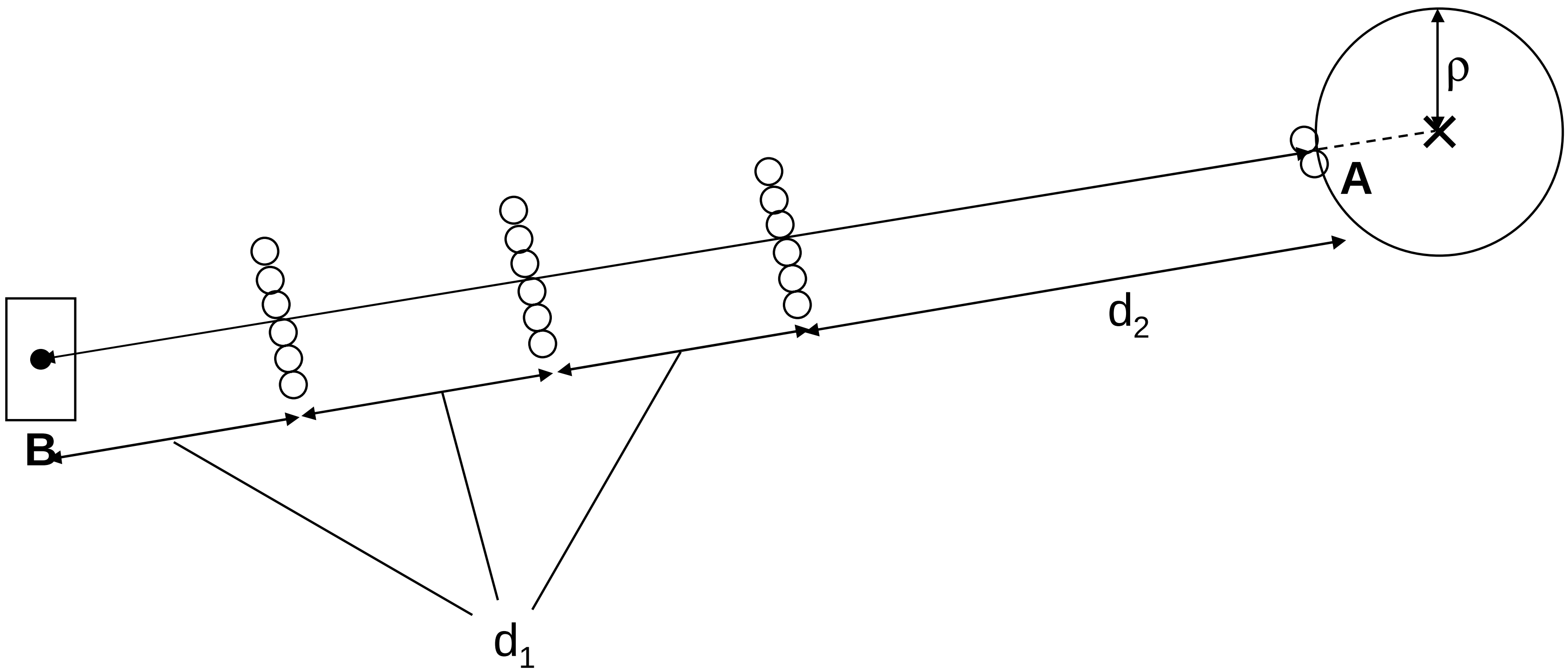
Take I



Take II

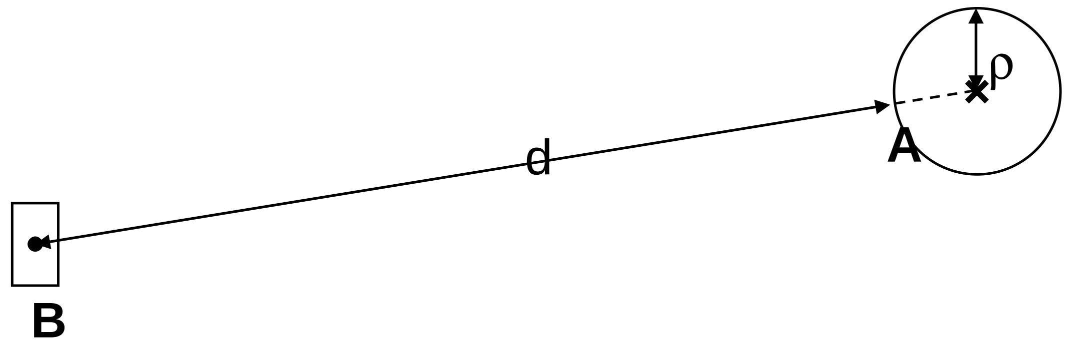


Take III



Need an alternative approach to bound lifetime …

Bounding Lifetime



- **Claim: At any instant in an active network:**
  - **There is a node that is sensing**
  - **There is a link of length d relaying bits at r bps**

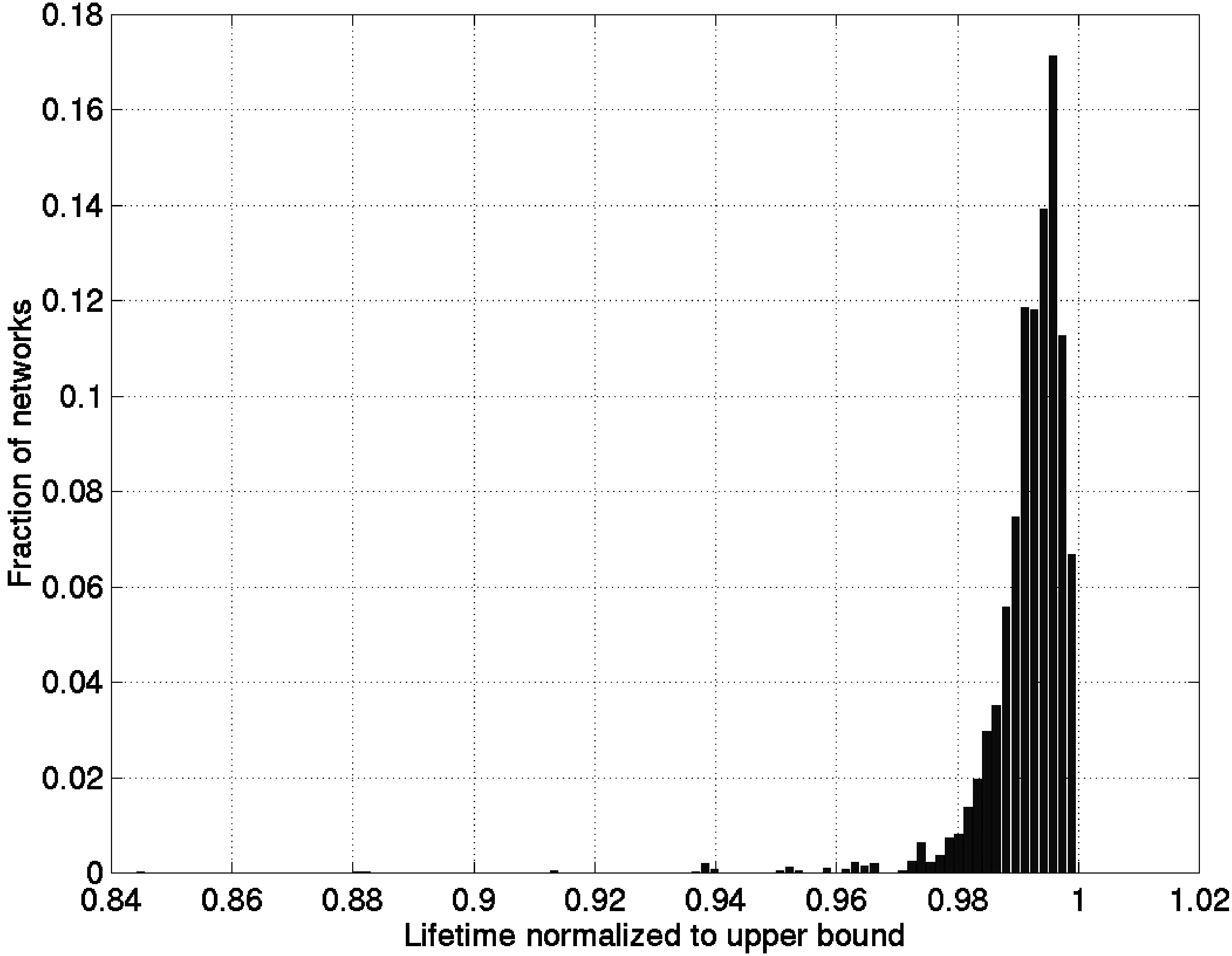
$$P_{\text{network}} \geq P_{\text{link}}(d) + P_{\text{sensing}} \quad \Rightarrow \quad P_{\text{network}} \geq \left(\frac{n\alpha_1}{n-1} \frac{d}{d_{char}} - \alpha_{12} \right) r + \alpha_3 r$$

- **If the network lifetime is T_{network}, then:**

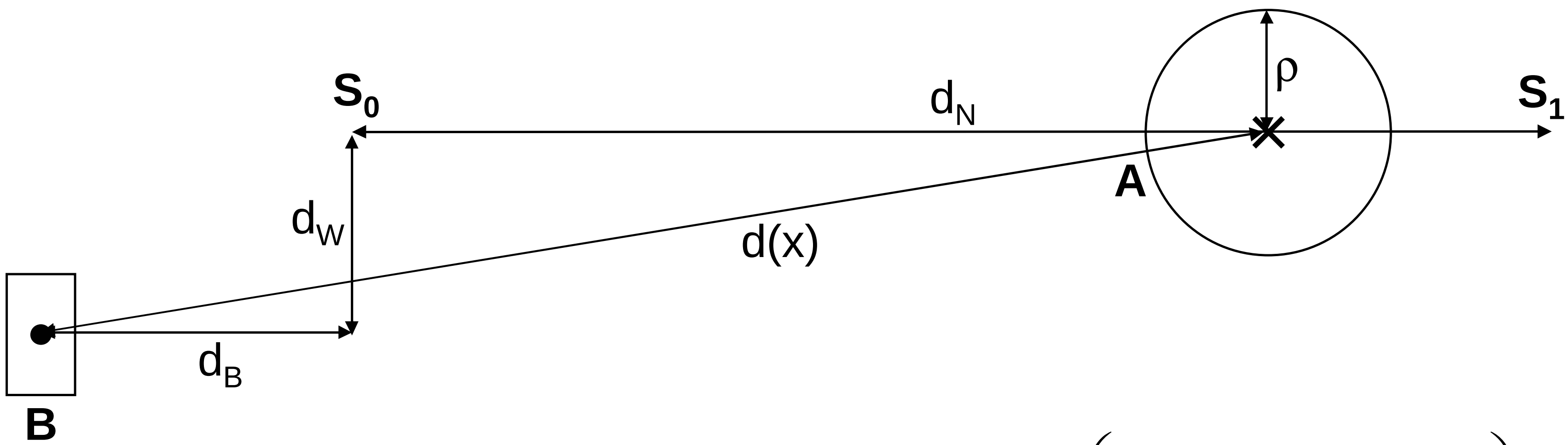
$$\sum_{i=1}^{N} E_i \geq \left\{ \left(\frac{n\alpha_1}{n-1} \frac{d}{d_{char}} - \alpha_{12} \right) r + \alpha_3 r \right\} T_{network}$$

$$T_{network} \leq \frac{N.E}{\left(\frac{n\alpha_1}{n-1} \frac{d}{d_{char}} - \alpha_{12} + \alpha_3 \right) r}$$

Simulation Results



Source Moving Along A Line

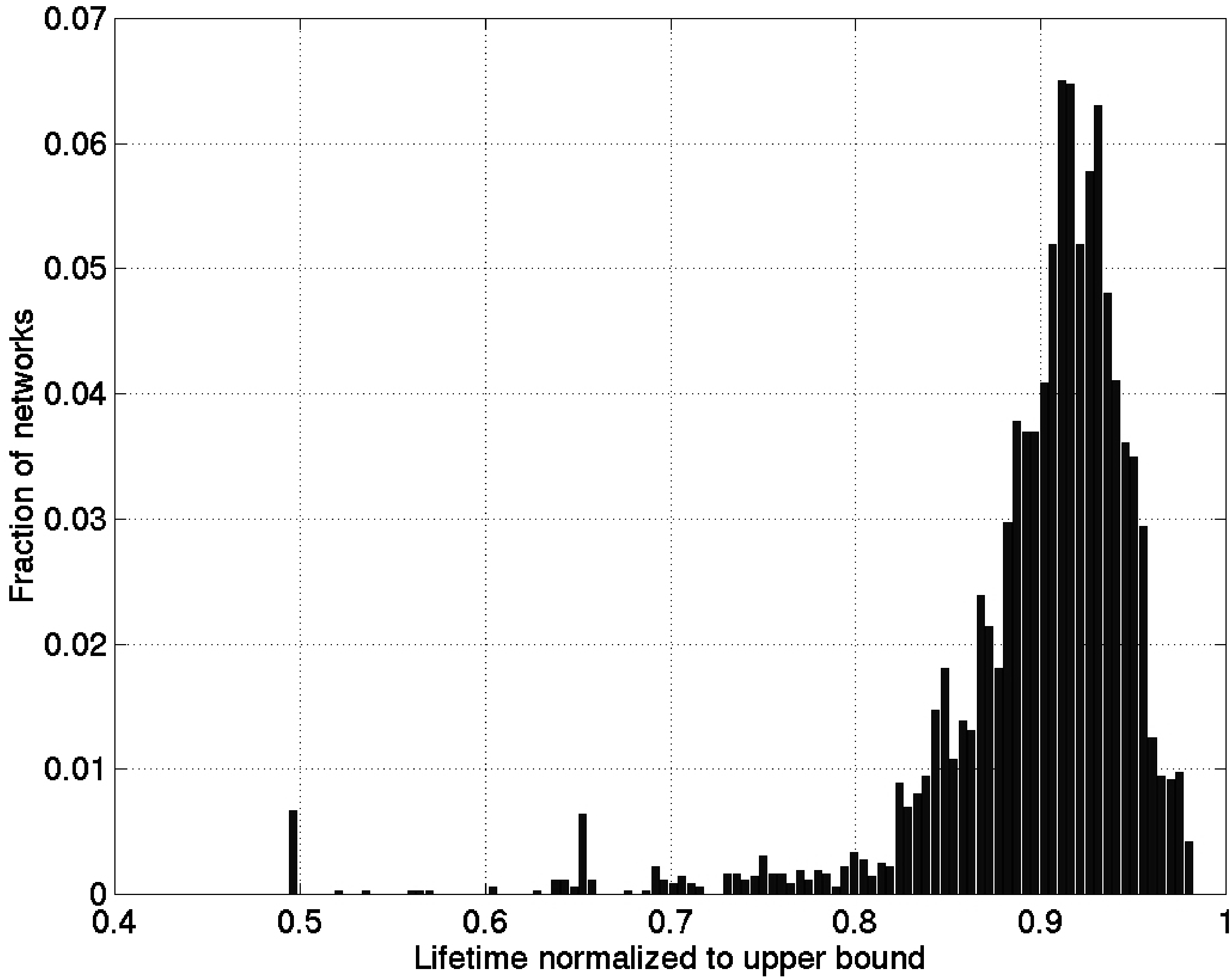


$$P_{\text{network}}(x) \geq P_{\text{link}}(d(x)) + P_{\text{sensing}} \Rightarrow P_{\text{network}}(x) \geq \left(\frac{n\alpha_1}{n-1} \frac{d(x)}{d_{char}} - \alpha_{12} \right) r + \alpha_3 r$$

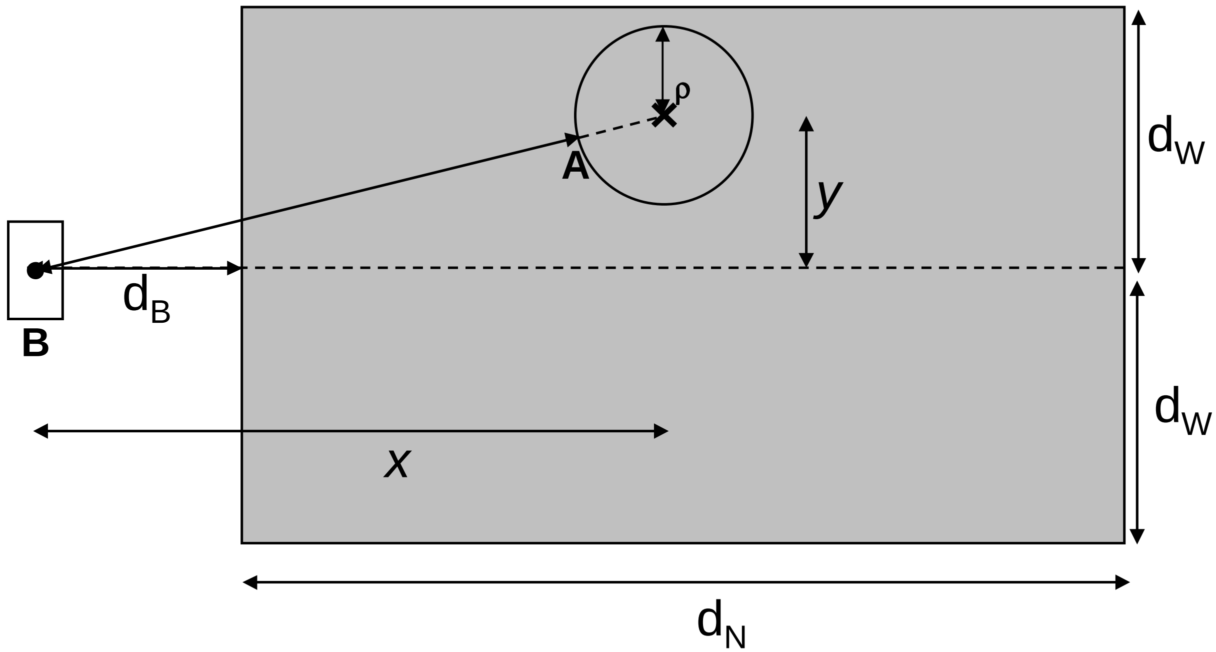
$$E(P_{\text{network}}) \geq \int_{x=d_B}^{x=d_B+d_N} P_{\text{network}}(x) l_{\text{source}}(x) dx$$

$$T_{network} \leq \frac{N.E}{\frac{n\alpha_1}{(n-1)d_{char}} \left(\frac{d_1 d_2 - d_3 d_4 + d_W^2 \ln\left(\frac{d_1 + d_2}{d_3 + d_4} \right)}{2d_N} - \rho \right) r}$$

Simulation Results



Source in a Rectangular Region



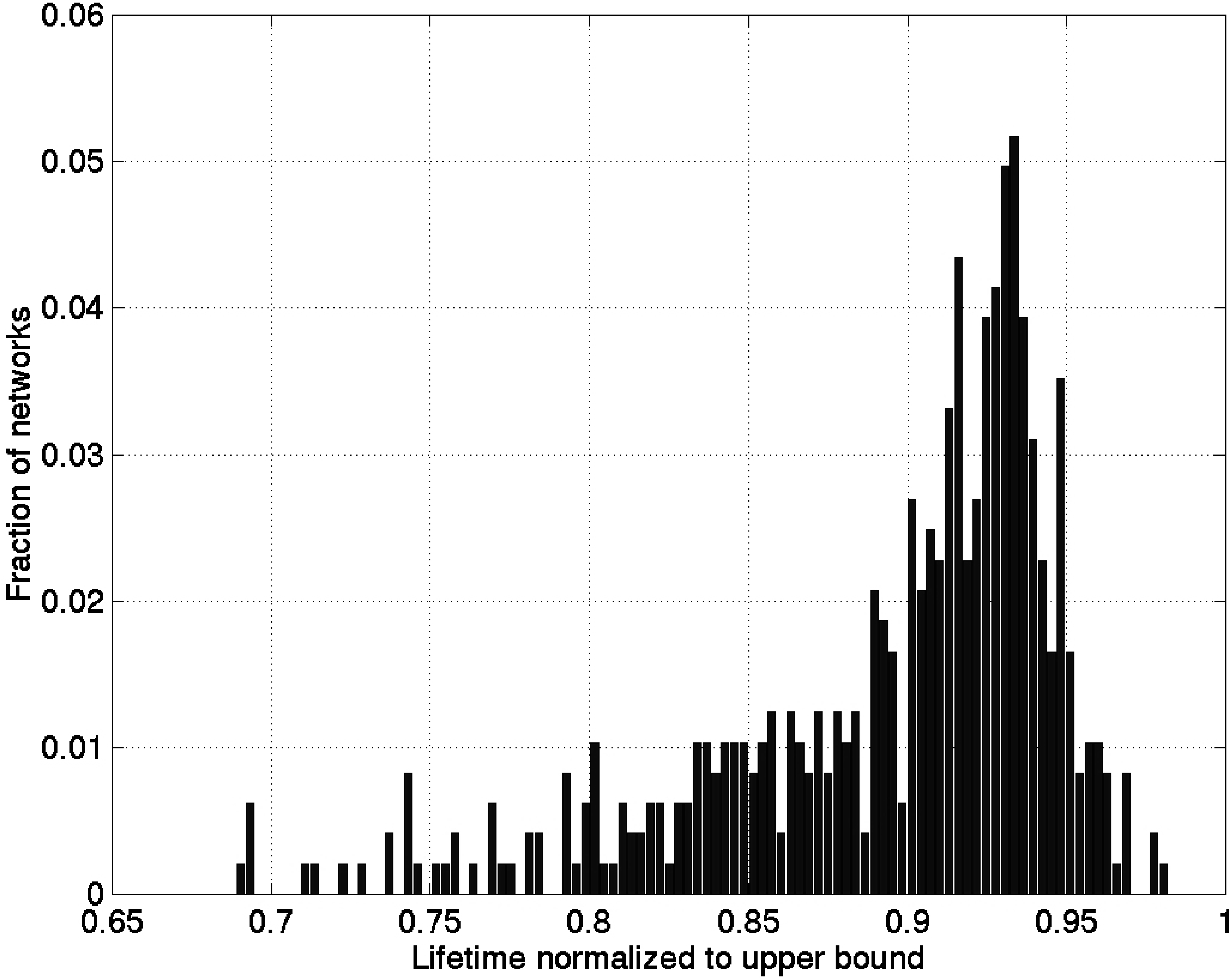
$$E(P_{\text{network}}) \geq \int_{x=d_B}^{x=d_B+d_N} \int_{y=-d_W}^{y=d_W} P_{\text{network}}(x,y)\, l_{\text{source}}(x,y)\, dx\, dy$$

$$T_{network} \leq \frac{N.E}{\frac{n}{n-1}\alpha_1 \frac{d_{rect}}{d_{char}} r}$$

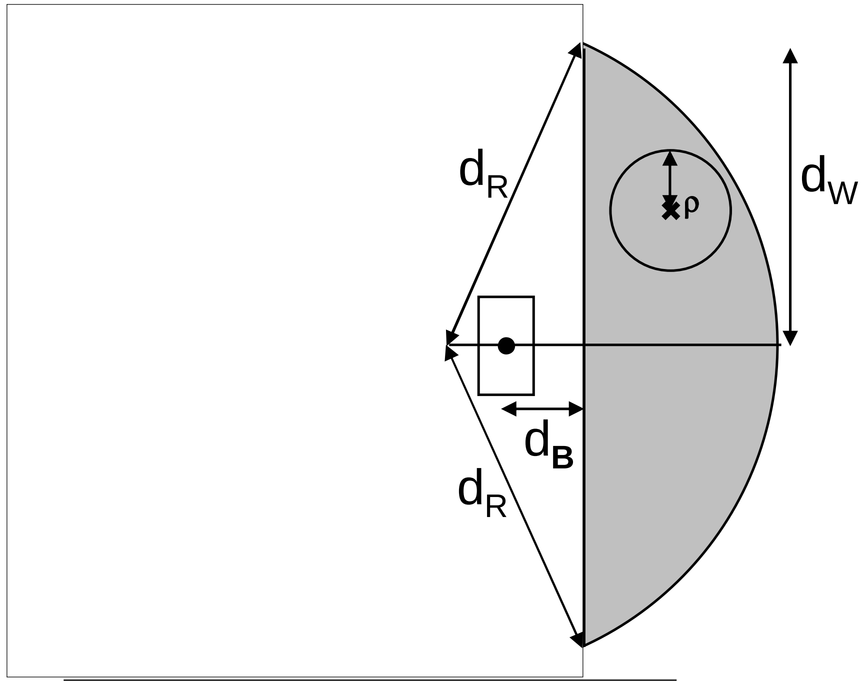
1000 node network, 2 J on a node has the potential to report finite velocity tank intrusions in a sq. km, a km away for more than 7 years!

$$d_{rect} = \frac{1}{12 d_N d_W}\left[4d_W(d_1 d_2 - d_3 d_4) + 2d_W^3 \ln\left(\frac{d_1+d_2}{d_3+d_4}\right) + d_3^3 \ln\left(\frac{d_4-d_W}{d_4+d_W}\right) + d_1^3 \ln\left(\frac{d_2+d_W}{d_2-d_W}\right)\right] - \rho$$

Simulation Results



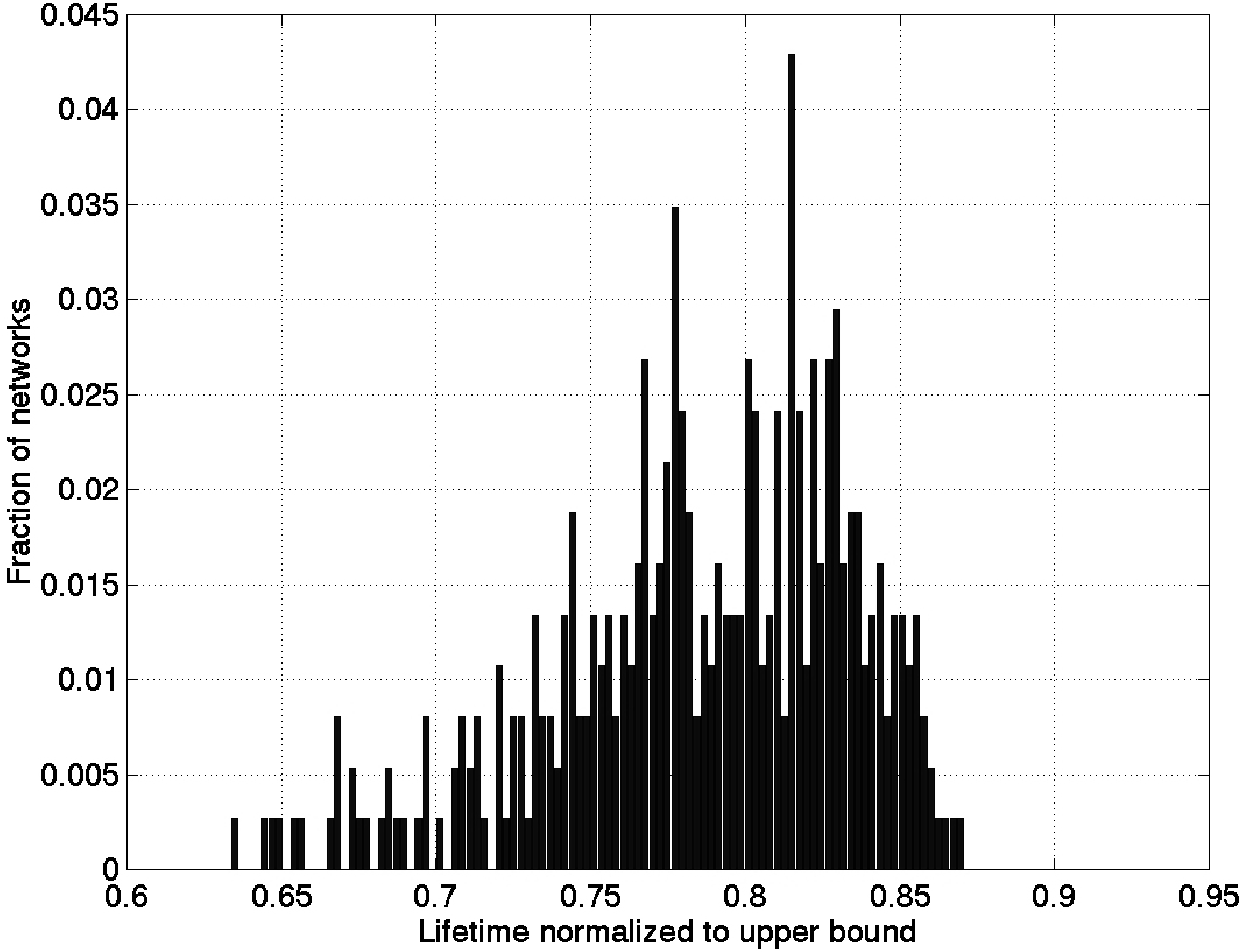
Source in a Semi-Circle



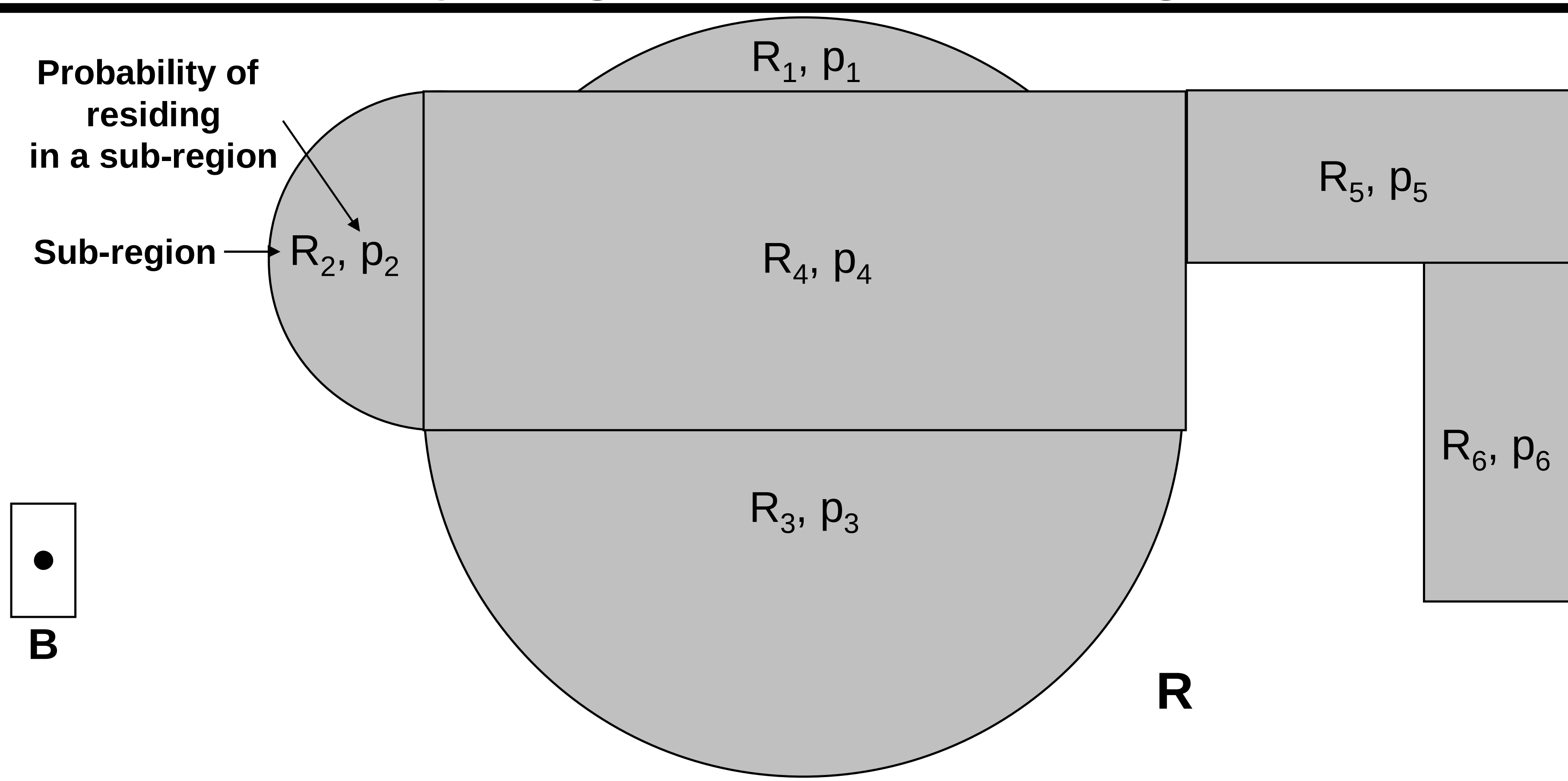
$$T_{network} \leq \frac{N.E}{\frac{n}{n-1}\alpha_1 \frac{d_{\text{sector}}}{d_{char}} r}$$

$$d_{\text{sector}} = \frac{2\theta d_R^3 - d_B d_R d_W - d_B^3 \ln\left(\frac{d_R + d_W}{d_B}\right)}{3(\theta(d_B^2 + d_W^2) - d_B d_W)} - \rho \quad \Rightarrow \quad d_{\text{semi-circle}} = \frac{2}{3} d_R - \rho$$

Simulation Results



Bounding Lifetime for Sources in Arbitrary Regions: Partitioning Theorem



Partitioning Relation:

$$T_{network}(R) \leq \left(\sum_{j=1}^{P} \frac{p_j}{T(R_j)} \right)^{-1}$$

Lifetime bound for region R_j (applies to $T(R_j)$)

Work completed subsequently …

- Factoring in topology
- Factoring in source movement
- Factoring in aggregation:
  - Flat aggregation
  - 2-step hierarchical
- Non-constructive approaches don't seem to work here
- Bounds derived by actual construction of the optimal strategy
- Strategy (and hence bound) can be derived in polynomial time

Summary

- Maximizing network lifetime is a key challenge in wireless sensor networks
- Using simple arguments based on minimum-energy relays and energy conservation, it is possible to derive tight or near-tight bounds on the lifetime of sensor networks
- It is possible to derive extremely sophisticated bounds that factor in the exact graph topology, source movement and aggregation